Prolog
Programming in Logic

Lecture #8

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Today's discussion

Videos

- Sudoku
- Constraints

Q: What are the extra-logical equalities?

A: So you know to avoid them:

```
x(A) == x(A)    -- Equivalence. x(A) == x(B) fails.
  D \== 1       -- Not equivalent
Exp1 =:= Exp2   -- E1 is Exp1, E1 is Exp2
Exp1 =\= Exp2   -- \+ E1 is Exp1, E1 is Exp2
   A =@= B      -- variants (<Equivalence >Unification).
     :=         -- ? defined operator, no function
     <=         -- ? undefined (use =<)
unify_with_occurs_check(A,B) -- should be an infix op.
```

```
foo :- ..., \+ Term, ...
```

Q: Is [a,b,c|A]-B a difference list?

A: If it's a quiz: answer is NO.

A: If it's a CS conversation, you could reason if:
A = [d,e,f|B], X = [a,b,c|A]-B
means that [a,b,c|A]-B *is* a difference list.

Although, similar is X an integer? yes if X=7 ...

Q: When is Prolog not ML-like?

ML factorial:

```
fun fact 0 = 1
  | fact n = n * fact (n - 1)

f = fact 5
```

Prolog factorial:

```
fact(1,1).
fact(N,Fact) :- N > 1, M is N-1, fact(M,Mfact), Fact is N * Mfact.

-? fact(5,F).
```

For a procedure intended to be used deterministically, with ground arguments, there is very little difference apart from syntax.

Type inference is a very clever bit of ML, while Prolog is typeless.

Q: When is Prolog not ML-like?

```
ML reverse list:
fun reverse [] = []
  | reverse (x::xs) = (reverse xs) @ [x]

l = reverse([1,2,3,4])

Prolog reverse list:
reverse([],[]).
reverse([X|XS],L) :- reverse(XS,XSrev), append(XSrev,[X],L).

?- reverse([1,2,3,4],L).
?- reverse(X, [4,3,2,1]). -- almost.
?- reverse([1,2,X,4],L).
?- reverse([1,2,X,Y],[4,3,2,A]).
```

Q: When is Prolog not ML-like?

Ultimately you can write a unification function in any language, define a data structure representing relations, and create a backtracking algorithm.

At that point you have 'Prolog' embedded in your language of choice.

Or you can modify Prolog to support higher-order functions...

I.e. implement the relational calculus in ML, or the functional calculus in Prolog.

Q: Last Call Optimisation

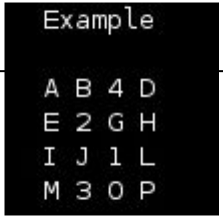
...a space optimization technique, which applies when a predicate is determinate at the point where it is about to call the last goal in the body of a clause.[1]

```
    last([L],L).
    last([_|T],L) :- last(T,L).
```

```
What about:
    foo(_,hello).
    foo(I, W) :- I > 10, J is I - 1, foo(J, W).
```

[1]Sicstus user manual

Sudoku 4x4 (video)



```
Example

A B 4 D
E 2 G H
I J 1 L
M 3 O P
```

```
range([]).
range([H|T]) :- range(1,5,H), range(T).

range(X,_,X).
range(X,Y,Next) :- N is X+1, N < Y, range(N,Y,Next).

diff(A,B,C,D) :- A=\=B, A=\=C, A=\=D, B=\=C, B=\=D, C=\=D.

rows([A,B,C,D,E,F,G,H,I,J,K,L,M,N,O,P]) :- diff(A,B,C,D), diff(E,F,G,H), diff(I,J,K,L), diff(M,N,O,P).

cols([A,B,C,D,E,F,G,H,I,J,K,L,M,N,O,P]) :- diff(A,E,I,M), diff(B,F,J,N), diff(C,G,K,O), diff(D,H,L,P).

boxes([A,B,C,D,E,F,G,H,I,J,K,L,M,N,O,P]) :- diff(A,B,E,F), diff(C,D,G,H), diff(I,J,M,N), diff(K,L,O,P).

sudoku(L) :- range(L), rows(L), cols(L), boxes(L).
```

```
[trace]  ?- sudoku([A,B,4,D,E,2,G,H,I,J,1,L,M,3,O,P]).
   Call: (8) sudoku([_1472, _1478, 4, _1490, _1496, 2, _1508, _1514|...]) ? creep
   Call: (9) range([_1472, _1478, 4, _1490, _1496, 2, _1508, _1514|...]) ? skip
   Exit: (9) range([1, 1, 4, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 3, 1, 1]) ? creep
   Call: (9) rows([1, 1, 4, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 3, 1, 1]) ? creep
   Call: (10) diff(1, 1, 4, 1) ? creep
   Call: (11) 1=\=1 ? creep
   Fail: (11) 1=\=1 ? creep
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   Fail: (9) rows([1, 1, 4, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 3, 1, 1]) ? creep
   Redo: (9) range([1, 1, 4, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 3, 1, _1562]) ? creep
   Exit: (9) range([1, 1, 4, 1, 1, 2, 1, 1, 1, 1, 1, 1, 1, 3, 1, 2]) ?
```

Sudoku 4x4 (video)

```
range([]).
range([H|T]) :- range(1,5,H), range(T).

range(X,_,X).
range(X,Y,Next) :- N is X+1, N < Y, range(N,Y,Next).

diff(A,B,C,D) :- A=\=B, A=\=C, A=\=D, B=\=C, B=\=D, C=\=D.

rows([A,B,C,D,E,F,G,H,I,J,K,L,M,N,O,P]) :- diff(A,B,C,D), diff(E,F,G,H), diff(I,J,K,L), diff(M,N,O,P).

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Sudoku 4x4 (video)

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Sudoku 4x4 (video)

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range([H|T]) :- range(1,5,H), range(T).

range(X,_,X).
range(X,Y,Next) :- N is X+1, N < Y, range(N,Y,Next).

diff(A,B,C,D) :- A=\=B, A=\=C, A=\=D, B=\=C, B=\=D, C=\=D. -- arithmetic not equals

rows([A,B,C,D,E,F,G,H,I,J,K,L,M,N,O,P]) :- diff(A,B,C,D), diff(E,F,G,H), diff(I,J,K,L), diff(M,N,O,P).

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```
[trace]  ?- sudoku([A,B,4,D,E,2,G,H,I,J,1,L,M,3,O,P]).
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```

Sudoku 4x4 (video)

```
Video efficiency improvement

diff(A,B,C,D) :- perm([1,2,3,4],[A,B,C,D]).

rows([A,B,C,D,E,F,G,H,I,J,K,L,M,N,O,P]) :- diff(A,B,C,D), diff(E,F,G,H), diff(I,J,K,L), diff(M,N,O,P).

cols([A,B,C,D,E,F,G,H,I,J,K,L,M,N,O,P]) :- diff(A,E,I,M), diff(B,F,J,N), diff(C,G,K,O), diff(D,H,L,P).

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sudoku(L) :- rows(L), cols(L), boxes(L).
```

```
?- sudoku([A,B,4,D,
           E,2,G,H,
           I,J,1,L,
           M,3,O,P]).
```

Sudoku 4x4 (video)

```
Video efficiency improvement

diff(A,B,C,D) :- perm([1,2,3,4],[A,B,C,D]).

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sudoku(L) :- rows(L), cols(L), boxes(L).
```

```
?- sudoku([A,B,4,D,
           E,2,G,H,
           I,J,1,L,
           M,3,O,P]).
```

Prolog vs ML... Imperative program ?:

```
function sudoku(board) {
    answer = make_board(board)
    if rows(answer) and cols(answer) and boxes(answer)
    then
        return answer
    else
        return fail
```

Sudoku 4x4 (video)

```
Video efficiency improvement

diff(A,B,C,D) :- perm([1,2,3,4],[A,B,C,D]).

rows([A,B,C,D,E,F,G,H,I,J,K,L,M,N,O,P]) :- diff(A,B,C,D), diff(E,F,G,H), diff(I,J,K,L), diff(M,N,O,P).

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sudoku(L) :- rows(L), cols(L), boxes(L).
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```
?- sudoku([A,B,4,D,
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Prolog vs ML... Imperative program ?:

```
function sudoku(board) {
        answer = make_board(board)
        if rows(answer) and cols(answer) and boxes(answer)
        then
                return answer
        else
                return fail
                            -- unification and backtracking ???
```

Sudoku 9x9 (Lecture #4)

```
% take(L1,X,L2) succeeds if output list L2 is input list L1 minus element X
take([H|T],H,T).
take([H|T],R,[H|S]) :- take(T,R,S).

% perm(L1,L2) succeeds if output list L2 is a permutation of input list L1
perm([],[]).
perm(A,[R|S]) :- take(A,R,P), perm(P,S).

% gen_digits(L) succeeds if L is a permutation of [1,2,3,4,5,6,7,8,9]
gen_digits(L) :- perm([1,2,3,4,5,6,7,8,9],L).
```

SUDOKU 9x9

```
puzzle1 :- RowA = [5,3,_,_,7,_,_,_,_],
           RowB = [6,_,_,1,9,5,_,_,_],
           RowC = [_,9,8,_,_,_,_,6,_],
           RowD = [8,_,_,_,6,_,_,_,3],
           RowE = [4,_,_,8,_,3,_,_,1],
           RowF = [7,_,_,_,2,_,_,_,6],
           RowG = [_,6,_,_,_,_,2,8,_],
           RowH = [_,_,_,4,1,9,_,_,5],
           RowI = [_,_,_,_,8,_,_,7,9],
           gen_digits(RowA),
           gen_digits(RowB),
           gen_digits(RowC),
           gen_digits(RowD),
           gen_digits(RowE),
           gen_digits(RowF),
           gen_digits(RowG),
           gen_digits(RowH),
           gen_digits(RowI),
           Test fixed numbers,
           Test the ‘boxes’,
           Test the ‘columns’,
           Print Solution.
```

```
solve([5,3,_,_,7,_,_,_,_],
      [6,_,_,1,9,5,_,_,_],
      [_,9,8,_,_,_,_,6,_],
      [8,_,_,_,6,_,_,_,3],
      [4,_,_,8,_,3,_,_,1],
      [7,_,_,_,2,_,_,_,6],
      [_,6,_,_,_,_,2,8,_],
      [_,_,_,4,1,9,_,_,5],
      [_,_,_,_,8,_,_,7,9]
     ).
```

```
solve(A,B,C,D,E,F,G,H,I) :- gen_digits(A),
                            test_cols([A,B,C,D,E,F,G,H,I]),
                            gen_digits(B),
                            test_cols([A,B,C,D,E,F,G,H,I]),
                            gen_digits(C),
                            test_boxes(A,B,C),
                            gen_digits(D),
                            test_cols([A,B,C,D,E,F,G,H,I]),
                            gen_digits(E),
                            test_cols([A,B,C,D,E,F,G,H,I]),
                            gen_digits(F),
                            test_cols([A,B,C,D,E,F,G,H,I]),
                            test_boxes(D,E,F),
                            gen_digits(G),
                            test_cols([A,B,C,D,E,F,G,H,I]),
                            gen_digits(H),
                            test_cols([A,B,C,D,E,F,G,H,I]),
                            gen_digits(I),
                            test_boxes(G,H,I),
                            test_cols([A,B,C,D,E,F,G,H,I]).
```

Q: Is our countdown mod iterative deepening?

A: ...

Q: Accumulating the Path to a Solution

```
choose(0, L, [], L).
choose(N, [H|T], [H|R], S) :- N > 0, N2 is N-1, choose(N2, T, R, S).
choose(N, [H|T], R, [H|S]) :- N > 0, choose(N, T, R, S).
```

Q: Accumulating the Path to a Solution

```
1. choose(0, L, [], L).
2. choose(N, [H|T], [H|R], S) :- N > 0, N2 is N-1, choose(N2, T, R, S).
3. choose(N, [H|T], R, [H|S]) :- N > 0, choose(N, T, R, S).
```

Step 1. Assign 'number' to each of the clauses in the procedure.

Q: Accumulating the Path to a Solution

```
1. choose(0, L, [], L).
2. choose(N, [H|T], [H|R], S) :- N > 0, N2 is N-1, choose(N2, T, R, S).
3. choose(N, [H|T], R, [H|S]) :- N > 0, choose(N, T, R, S).
```

Step 1. Assign 'number' to each of the clauses in the procedure.

Step 2. Add 2 arguments (+PathIn, -PathOut) to every (non-deterministic) relation, for the accumulated path so far and the path when this clause succeeds, e.g.:

```
choose(0,L,[],L).
```

becomes:

```
choose(0,L,[],L, PathIn, [1|PathIn]).
```

When we initially call 'choose' we will set 'PathIn' to [], and expect the completed path in PathOut:

```
?- choose(2,[a,b,c,d,e],Chosen,Remaining,[],Path).
```

Q: Accumulating the Path to a Solution

```
1. choose(0,L,[],L, PathIn, [1|PathIn]).
2. choose(N, [H|T], [H|R], S) :- N > 0, N2 is N-1, choose(N2, T, R, S).
3. choose(N, [H|T], R, [H|S]) :- N > 0, choose(N, T, R, S).
```

Step 1. Assign 'number' to each of the clauses in the procedure.

Step 2. Add 2 arguments (+PathIn, -PathOut) to every (non-deterministic) relation, for the accumulated path so far and the path when this clause succeeds.

Step 3.

```
choose(N, [H|T], [H|R], S) :-
     N > 0, N2 is N-1,
     choose(N2, T, R, S).
```

becomes:

```
choose(N, [H|T], [H|R], S, PathIn, PathOut) :-
     N > 0, N2 is N-1,
     choose(N2, T, R, S,[2|PathIn],PathOut).
```

Q: Accumulating the Path to a Solution

```
choose(0,L,[],PathIn, [1|PathIn]).

choose(N, [H|T], [H|R], S, PathIn, PathOut) :-
     N > 0, N2 is N-1,
     choose(N2, T, R, S,[2|PathIn],PathOut).

choose(N, [H|T], R, [H|S], PathIn, PathOut) :-
     N > 0,
     choose(N, T, R, S,[3|PathIn],PathOut).
```

```
?- choose(2,[a,b,c,d,e],Chosen,Remaining,[],Path).

     Chosen = [a, b],                  Chosen = [a, e],
     Remaining = [c, d, e],   . . .    Remaining = [b, c, d, f, g],
     Path = [1, 2, 2]                  Path = [1, 2, 3, 3, 3, 2]
```

Path should be read 'backwards'

Q: Accumulating the Path to a Solution

```
choose(0,L,[],PathIn, [1|PathIn]).

choose(N, [H|T], [H|R], S, PathIn, PathOut) :-
     N > 0, N2 is N-1,
     choose(N2, T, R, S,[2|PathIn],PathOut).

choose(N, [H|T], R, [H|S], PathIn, PathOut) :-
     N > 0,
     choose(N, T, R, S,[3|PathIn],PathOut).
```

```
?- choose(2,[a,b,c,d,e],Chosen,Remaining,[],[1, 2, 3, 3, 3, 2]).
                                Chosen = [a, e],
                                Remaining = [b, c, d, f, g]
```

...DEMO ...

Q: Is our countdown mod iterative deepening?

```prolog
% solve(+N, +SolnAcc, -Soln) succeeds if Soln is a solution
%  to the NxN queens problem and is an extension of
%  the accumulated solution in SolnAcc.
% e.g. :- solve(10, [], Soln), draw(Soln).
solve(N, SolnAcc, SolnAcc) :- length(SolnAcc,N).
solve(N, SolnAcc, Soln) :-
    move(N, SolnAcc, Square),
    solve(N, [Square | SolnAcc], Soln).

% move(+N, +Soln, -Square) acts as a generator for
%  'safe' squares in the next row to those already
%  allocated in the partial solution in Soln.
move(N, [], sq(1, Col)) :- drange(N, Col).
move(N, [sq(I,J) | Rest], sq(Row, Col)) :-
    I < N, Row is I + 1, drange(N, Col), safe(sq(Row, Col), [sq(I,J)|Rest]).

% drange(+N,-X) acts as generator for X = N down to 1.
drange(N, N).
drange(N, X) :- M is N - 1, M > 0, drange(M, X).

% safe_square(+QueenSquare,+Square) succeeds if QueenSquare
%  does not attack Square (or vice-versa).
safe_square(sq(I, J), sq(X, Y)) :- I \== X, J \== Y,
    U1 is I - J, V1 is X - Y, U1 \== V1,
    U2 is I + J, V2 is X + Y, U2 \== V2.

% safe(+Square,+Soln) succeeds if Square does not attack
% any queens in the partial solution Soln.
safe(_, []).
safe(Square, [QueenSquare | L]) :-
    safe_square(QueenSquare, Square), safe(Square, L).
```

N-Queens

?- solve(10,[],Soln).

Q: Is our countdown mod iterative deepening?

```
% solve(+N, +SolnAcc, -Soln) succeeds if Soln is a solution
%  to the NxN queens problem and is an extension of
%  the accumulated solution in SolnAcc.
% e.g. :- solve(10, [], Soln), draw(Soln).
solve(N, SolnAcc, SolnAcc) :- length(SolnAcc,N).
solve(N, SolnAcc, Soln) :-
    move(N, SolnAcc, Square),
    solve(N, [Square | SolnAcc], Soln).

% move(+N, +Soln, -Square) acts as a generator for
%  'safe' squares in the next row to those already
%  allocated in the partial solution in Soln.
move(N, [], sq(1, Col)) :- drange(N, Col).
move(N, [sq(I,J) | Rest], sq(Row, Col)) :-
    I < N, Row is I + 1, drange(N, Col), safe(sq(Row, Col), [sq(I,J)|Rest]).

% drange(+N,-X) acts as generator for X = N down to 1.
drange(N, N).
drange(N, X) :- M is N - 1, M > 0, drange(M, X).

% safe_square(+QueenSquare,+Square) succeeds if QueenSquare
%  does not attack Square (or vice-versa).
safe_square(sq(I, J), sq(X, Y)) :- I \== X, J \== Y,
    U1 is I - J, V1 is X - Y, U1 \== V1,
    U2 is I + J, V2 is X + Y, U2 \== V2.

% safe(+Square,+Soln) succeeds if Square does not attack
% any queens in the partial solution Soln.
safe(_, []).
safe(Square, [QueenSquare | L]) :-
    safe_square(QueenSquare, Square), safe(Square, L).
```

N-Queens

?- solve(10,[],Soln).

DETERMINISTIC

N-Queens with path accumulation

```
solve(N, SolnAcc, SolnAcc, PathIn, [1|PathIn]) :- length(SolnAcc,N).
solve(N, SolnAcc, Soln, PathIn,PathOut) :-
   move(N, SolnAcc, Square,[2|PathIn],PathOut1),
   solve(N, [Square | SolnAcc], Soln, PathOut1, PathOut).

move(N, [], sq(1, Col), PathIn, PathOut) :- drange(N, Col, [1|PathIn], PathOut).
move(N, [sq(I,J) | Rest], sq(Row, Col), PathIn, PathOut) :-
   I < N, Row is I + 1,
   drange(N, Col, [2|PathIn], PathOut), safe(sq(Row, Col), [sq(I,J)|Rest]).

% drange(N,X) acts as generator for X = N down to 1.
drange(N, N, PathIn, [1|PathIn]).
drange(N, X, PathIn, PathOut) :-
      M is N - 1, M > 0,
      drange(M, X, [2|PathIn], PathOut).
```

N-Queens

?- solve(10,[],Soln,[],Path).

ANOTHER DEMO...

Hanoi with path accumulation

```
% Hanoi puzzle
% Towers A, B, C. Each a list of rings 1,2,3 (Head = top).

ok_move([A1|A],[],A,[A1]).
ok_move([A1|A],[B1|B],A,[A1,B1|B]) :- A1 < B1.

% move A -> B
move(state(A,B,C), state(AN,BN,C), PathIn, [1|PathIn]) :- ok_move(A,B,AN,BN).
% move A -> C
move(state(A,B,C), state(AN,B,CN), PathIn, [2|PathIn]) :- ok_move(A,C,AN,CN).
% move B -> A
move(state(A,B,C), state(AN,BN,C), PathIn, [3|PathIn]) :- ok_move(B,A,BN,AN).
% move B -> C
move(state(A,B,C), state(A,BN,CN), PathIn, [4|PathIn]) :- ok_move(B,C,BN,CN).
% move C -> A
move(state(A,B,C), state(AN,B,CN), PathIn, [5|PathIn]) :- ok_move(C,A,CN,AN).


hanoi(States, State_from, State_to, PathIn, PathOut) :-
       move(State_from,State_to, [1|PathIn], PathOut),
       nl,print(States).


hanoi(States, State_from, State_to, PathIn, PathOut) :-
       move(State_from, Next_state, [2|PathIn], PathOut1),
       \+ member(Next_state, States),
       hanoi([Next_state|States],Next_state,State_to, PathOut1, PathOut).


solve(Path) :- hanoi([],state([1,2,3],[],[]),state([],[],[1,2,3]), [], Path).
```

YET ANOTHER DEMO...

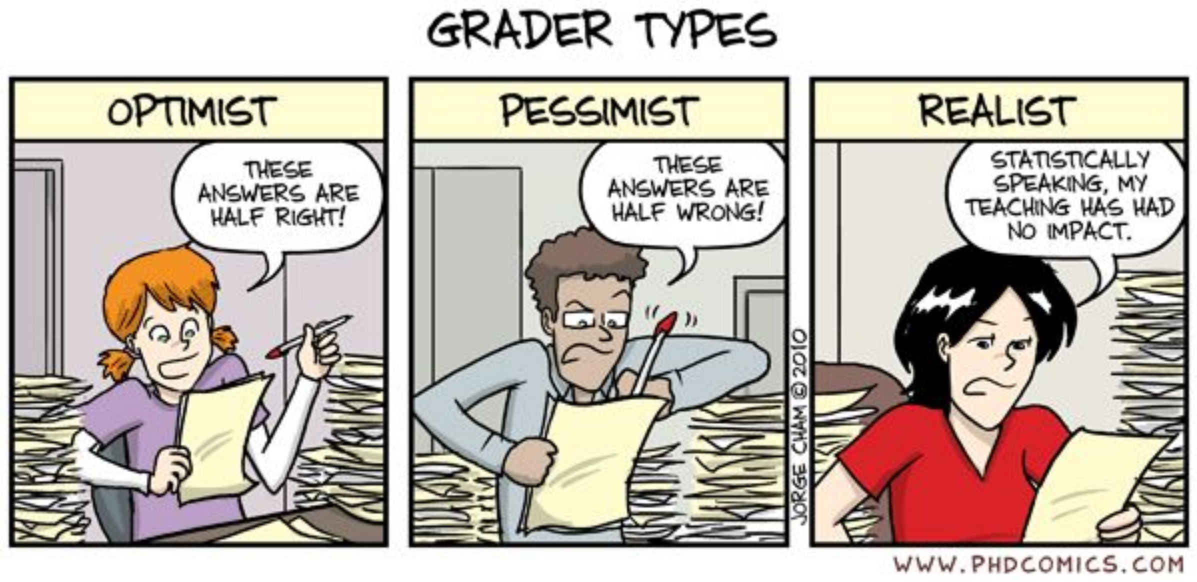
Hanoi with depth limit

```
% Hanoi puzzle
% Towers A, B, C. Each a list of rings 1,2,3 (Head = top).

ok_move([A1|A],[],A,[A1]).
ok_move([A1|A],[B1|B],A,[A1,B1|B]) :- A1 < B1.

% move A -> B
move(state(A,B,C), state(AN,BN,C), PathIn, [1|PathIn]) :- length(PathIn,N), N < Limit, ok_move(A,B,AN,BN).

 •    •    •



solve(Path, Limit) :- hanoi([],state([1,2,3],[],[]),state([],[],[1,2,3]), [], Path, Limit).
```

YET ANOTHER DEMO...

End... of... the... course...

I hope you found the format helpful - please fill out the feedback forms!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OPTIMIST
THESE ANSWERS ARE HALF RIGHT!
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